Dark Matter orbits intersecting dense Normal Matter objects

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I present a simple analysis of the orbits of dark matter particle or clump of particles should follow when crossing a dense normal matter object such as a planet or a star. This simple analysis should serve to correct existing publications and could be used a starting point for a more detailed analysis which will require better modeling of the dark matter and the dense normal matter object.

I. BACKGROUND

Dark Matter models are one of the existing proposals to explain galaxy level and cosmological level dynamics discrepancies if only matter and energy from the Standard Model and General Relativity is taken into account.

As such, this Dark Matter (DM onward) should have little or no electromagnetic, strong or weak interaction with Normal Matter (NM onward).

The only noticeable effects we can test are gravitational effects DM produces in NM.

Previous gravitational analysis known to the author focus on how dark matter distribution affects known observational data and only dark matter heating [1] in which bursts of NM out of galaxies also alter the distribution of DM to higher orbits.

As of version 3 of this paper it has come to the author knowledge of the Doctoral Thesis by Marina Cermeño Gavilán titled “Dark matter in dense astrophysical objects” [11] and referenced papers in it, which covers more extreme scenarios and theoretical interactions other than gravity and don’t cover this more simpler case.

Some articles found for general public “What Would Happen If You Became Dark Matter? (2017)”[2] and later “Que se passerait-il si la matière ordinaire qui nous compose était convertie en matière noire? (2018)” [3] present DM particles orbiting within dense objects following Kepler orbits, nevertheless that should not be the case if those hypothetical particles exists as I will show in this paper.

II.- GLOBAL ASSUMPTIONS

Since this is a simple approach there are some assumptions which align with what is assumed today for DM and dense NM bodies like asteroids, planets or stars:

- DM only interacts with NM via gravitation.
- DM is modeled as one indivisible distribution of mass (in particular it will be a point-like mass). We will refer to the DM particle.
- NM object will be modeled as a spherical symmetrical non rotating object hold together by its gravity in equilibrium by electromagnetic forces between the atoms.
- The regime of the study will be considered in the low energies so no relativistic effects are relevant (like motion of DM and NM is small, radius of NM object is big in comparison to the Schwarzschild radius…).
- The mass of the DM particle is very low in comparison with the mass of the NM object
- The DM particle is bounded to the NM object (it has not enough energy to escape to infinite) and there are no other massive objects that affect the analysis

III. INITIAL ANALYSIS

To simplify initial approach we will add an initial assumption that will be later be dropped which can be expressed like the DM doesn’t loose any energy from interaction with NM.

This initial analysis cover the case of a DM particle orbiting a NM object without crossing the NM object at any time.

In this case the DM orbit is an ellipse with one of the focus in the center of the NM object, this orbit is closed. That scenario is represented in Figure 1. where the NM object is the first circle.

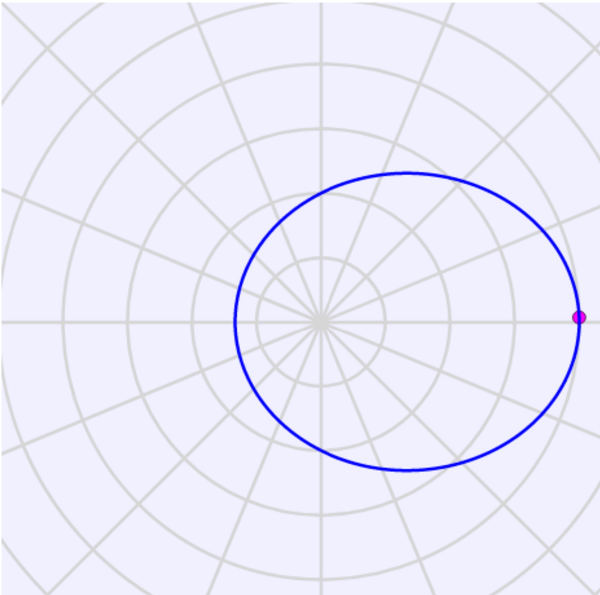


Figure 1: DM orbit not crossing NM Object

which follows a central force of the form:

$$f(r) = G\frac{Mm}{r^2} \quad \textit{for } r > \textit{NM object radius}$$

With M being the mass of the NM object, m being the mass of the DM object and G is the Gravitational Constant.

The second scenario is that in which the whole orbit of the DM particle is embedded into the NM body.

As a first step we will consider the NM body of uniform density.

In this case we need to take into account the Shell Theorem [4] and only the mass of the sphere centered in the NM object center up limited to the position of the DM particle really accounts for gravitation force, so the inverse square law does not apply and rather as Wikipedia says *"inside a solid sphere of constant density, the gravitational force within the object varies linearly with distance from the center, becoming zero by symmetry at the center of mass"*.

$$f(r) = \frac{4}{3}\pi m \rho G r \quad \textit{for } r < \textit{NM object radius}$$

Where ρ is the NM object density, m and G same as previous scenario.

That kind of force F=kr is the kind of force a spring exerts on an object and the resulting orbit for it is also an ellipse but this time with the center of the ellipse being the center of the NM object. That scenario is represented in Figure 2. where the NM object covers all the area represented.

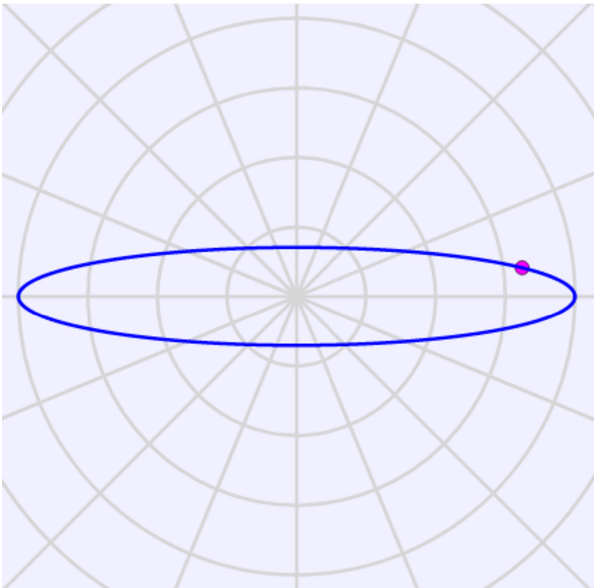


Figure 2: DM orbit fully inside uniform NM Object

The third and last scenario in this chapter is that in which we keep the NM object properties but part of the orbit of the DM particle is inside of the NM object while other part is outside of the NM object.

$$f(r) = \begin{cases} \frac{4}{3}\pi m \rho G r & \textit{for } r < \textit{NM object radius} \\ G\frac{Mm}{r^2} & \textit{for } r > \textit{NM object radius} \end{cases}$$

In this third scenario the central force field is not aligned to either isotropic oscillator or Kepler orbits, and as per Brentand's Theorem [5] in general (e.g. excluding circular orbits) the orbits will not be closed.

This third scenario will also be present if we consider the NM body made of layers of different density, or where the density varies as a function of depth, expecting to be higher in the center.

An example of such scenario is represented in the following figure in which the radius of the NM object has been set to 3.

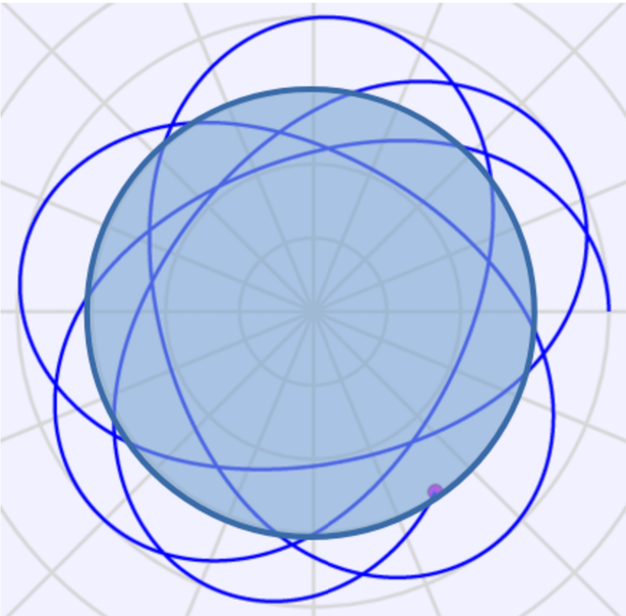


Figure 3: DM partially crossing uniform NM object

As we can see there will be a maximum height and maximum depth where the DM particle will be, deviating clearly from the initial version of the articles that triggered the creation of this paper. [2][3]

What will be the effect in a layered sphere?

It could be argued that due to differences in density, growing the deeper in the sphere, that will divert from the previous approach to more like a Keplerian orbit, nevertheless there are 2 ways to analyze that.

Theoretically, we can compare the effects of a Dirac δ of density at r=0 (Keplerian orbits) with that of a density that goes from a minimum (at r=NM object radius) to a maximum (at r=0). Knowing the radius of the NM object, the maximum density, the minimum density and the total mass, and assuming density increases or is kept equal as r decreases we can the derive the density distribution that is closer to that Dirac δ at r=0, being it a 2 layered distribution with maximum density in one inner layer and another outer layer of minimum density. We can the calculate the inter-layer radius R_{if}

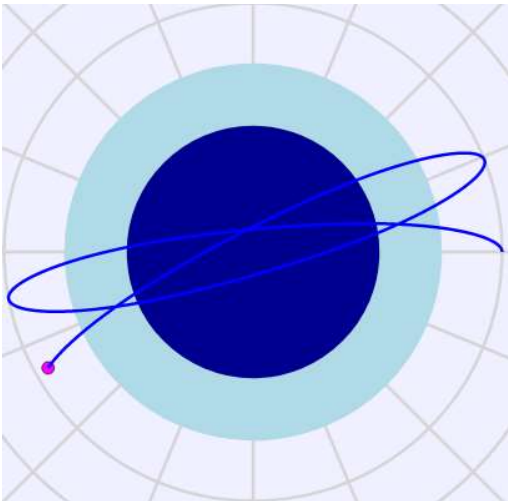
$$Total\,mass = M_T = 4/3\,\pi\,R_{if}^3\,\rho_{max} + 4/3\,\pi\,\rho_{min}\left(R^3 - R_{if}^3\right)$$

$$R_{if} = \sqrt[3]{\frac{M_T - 4/3\,\pi\,\rho_{min}\,R^3}{4/3\,\pi\,\rho_{max} - 4/3\,\pi\,\rho_{min}}}$$

For earth as per earthhow.com [12] the density of the different layers can be approximated to:

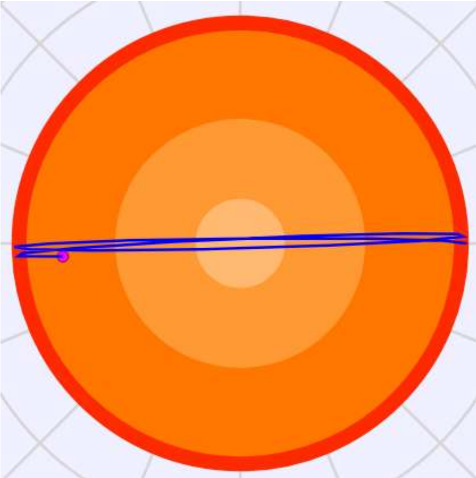
Crust:	2.5 g/cm3
Upper Mantle:	4g/cm3
Lower Mantle:	5g/cm3
Outer Core:	11g/cm3
Inner Core:	13g/cm3

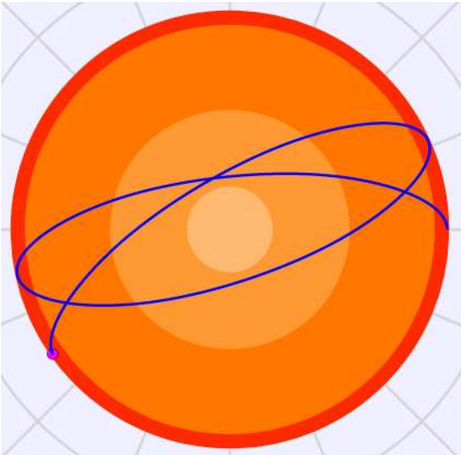
So with density varying between 13g/cm3 to 2.5g/cm3 and with a radius of 6371 km and total mass of $5{,}97\cdot 10^{24}$ we get that the best approximation to a Dirac δ of density would be where there are only two layers with a R_{if} at 4202 km, being this the closest to Keplerian orbit it can get.

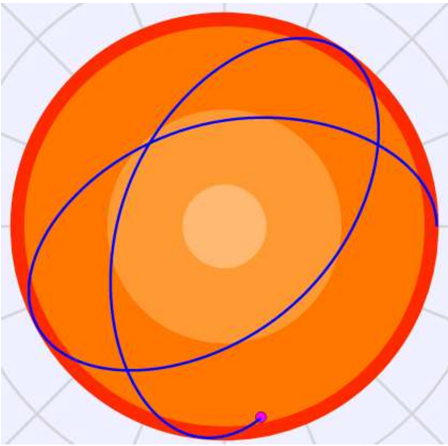


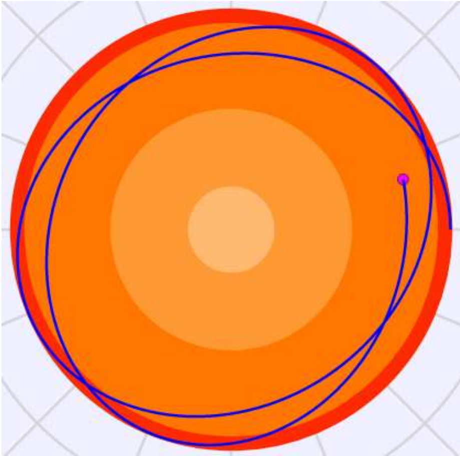
Alternative via simulation [13], we can create a layered model of constant densities within each layer, and sizes of each layer proportional to earth's and actually see the possible orbits.

For that purpose a different simulations can be run.

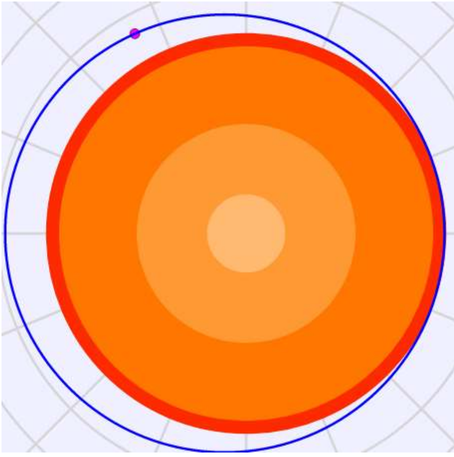








Only when there is no crossing with the NM object the orbit becomes a Keplerian closed orbit.



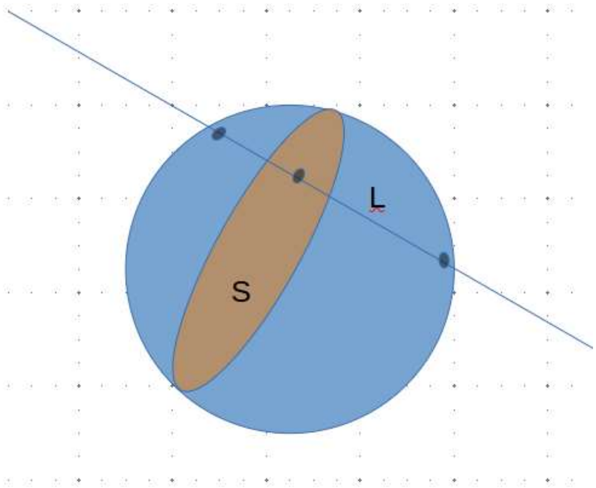
IV.- DYNAMICAL FRICTION EFFECTS

The previous analysis is based on the assumption that no energy is transferred from the DM particle to the NM object or the surrounding space-time.

It is known that via gravitation objects can exchange energy, like the slingshot effect used by space probes to get or loose kinetic energy assisted by the gravity of an orbiting planet or moon.

In this particular case I will cover the effect called Dynamical friction [6] which as in the case of Dark Matter topic has been studied on solar system scales or bigger, but not the cases covered in this paper, at least known to the author.

A DM particle that crosses NM object should loose energy in this form, giving part of the DM kinetic energy to compress the NM around it and that work is ultimately converted to heat in the NM.



Assuming that the work done in an object of mass m by increasing the pressure and keeping temperature constant can be approximated as:

$$W \approx -\frac{M k}{2\rho}(P_f^2 - P_i^2) = -\frac{M k}{2\rho}(\Delta P^2 + \Delta P P_i)$$

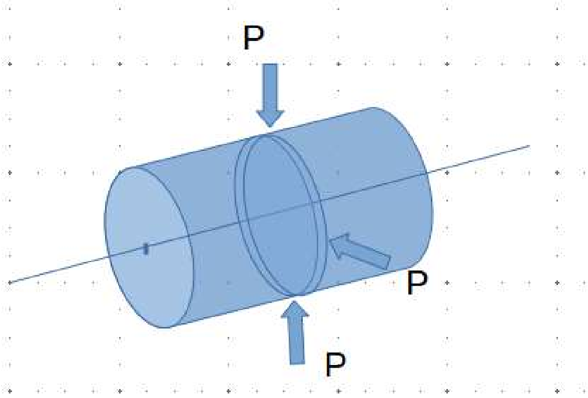
(taken from stackexchange question [7]) where $\Delta P = P_f - P_i$

We can think about calculating the work done in any section of the NM object perpendicular to the trajectory of the DM object, assuming constant density, compressibility, and low speed, then only we can consider that ΔP, Pi are the values to integrate through the cross sections as the DM particle passes.

$$dW \approx -\frac{k}{2\rho}(\Delta P^2 + \Delta P P_i) dM$$

$$W \approx \int_L \int_S -\frac{k}{2\rho}(\Delta P^2 + \Delta P P_i) \rho \, dS \, dl$$

As an initial approximation we can consider what is the dynamical friction produced by a cylinder of NM of radius R, density ρ, compressibility k with every particle in it affected by an environmental or self-pressure Pi (constant or changing only on the depth within the length of the cylinder) that is traversed by a DM particle of mass m



Assuming P_i= Constant and to calculate the work done due to the increment of pressure on a dl ring of mass dm we have:

$$dW \approx -\frac{k}{2\rho}(\Delta P^2 + \Delta P P_i) dM$$

with

$$dM = 2\pi r \rho \, dr \, dl$$

In general the hydro-static equilibrium for a column of liquid can be written for constant density as:

$$dP = -\rho(P) g(h) dh = -\rho g(h) dh$$

with g being the acceleration at that point, if instead of height we use distance r from a central acceleration due to a mass m of DM we can rewrite the above as:

$$dP = -\rho g(h) dh = -\rho G \frac{m}{r^2} dr$$

Since we want to check the increment ΔP for the ring will be when the DM particle is at the ring center and we can calculate the hydro static equilibrium due only to that mass with:

$$d\Delta P = -\rho G \frac{m}{r^2} dr$$

Integrating the differential then ΔP for the ring of radius r is

$$\Delta P=\rho G\frac{m}{r}$$

so now the differential of work is:

$$dW\approx-\frac{k}{2\rho}[(\rho G\frac{m}{r})^2+(\rho G\frac{m}{r})P_i]2\pi r\rho dr\,dl$$

which can be simplified to:

$$dW\approx-\pi k\rho G(\rho G\frac{m^2}{r}.+mP_i)dr\,dl$$

Assuming in the vicinity ε of the DM particle is empty space (as it is for distances below the inter atomic distances) we can calculate the work done on a dl section of the cylinder from that empty space to a radius R integrating the rings on dr, we then get:

$$\frac{dW}{dl}\approx-\pi k\rho G(\rho G m^2(\ln(R)-\ln(\varepsilon))+mP_i(R-\varepsilon))$$

Also since the work done compressing the slice comes from DM kinetic energy loss we can estimate the energy and its lost as:

$$E_k = ½\, m\, v^2$$

As such then the differential lost is given by:

$$dE_k=\frac{1}{2}m2v\,dv=mvdv$$

Equating both expressions we get that:

$$mvdv\approx-\pi k\rho G(\rho G m^2(\ln(R)-\ln(\varepsilon))+mP_i(R-\varepsilon))dl$$

$$vdv\approx-\pi k\rho G(\rho G m(\ln(R)-\ln(\varepsilon))+P_i(R-\varepsilon))dl$$

So after a length of cylinder L we get

$$v_f^2-v_o^2\approx-2\pi k\rho G(\rho G m(\ln(R)-\ln(\varepsilon))+P_i(R-\varepsilon))L$$

Characteristics

This above formula has the following characteristics:

* It is linearly dependent on the length of the cylinder, its density and its compressibility.

* It is “small” in the sense that it depends on the gravitational constant G

* It has 2 different parts, one depends linearly on the mass of the DM object, only logarithmic on the radius of the cylinder and is double dependent on the gravitational constant G. This term is only significant for large masses. e.g. could be relevant for a black hole of the size of an atom with mass around 10^17Kg.

This part satisfies the velocity squared units, being the logarithm dimensionless:

$$\frac{m^2}{Kg\,m\,s^{-2}}(\frac{Kg}{m^3})^2(\frac{m^3}{Kg\,s^2})^2Kg\,m=\frac{m^2}{s^2}$$

* The second part is independent of the mass of the DM object and has linear dependency on the mass encircling the DM object and the initial pressure to which is subjected to. This term is significant for low mass DM objects that cross large NM objects which already are in hydro-static equilibrium and high pressure under the surface.

This second part also satisfies the velocity squared units:

$$\frac{m^2}{Kg\,m\,s^{-2}}(\frac{Kg}{m^3})(\frac{m^3}{Kg\,s^2})(\frac{Kg\,m\,s^{-2}}{m^2})m\,m=\frac{m^2}{s^2}$$

* This equation is only physically valid until v_f is zero. And it is based on length traversed.

This formula can be compared with the one in Wikipedia for which also there is an inverse relationship with velocity (cubed) but in this case, the formula is bounded.

* For already gravitationally bounded DM objects, as the DM particle looses energy it

should end up in (or quite near) the center of gravity of the NM object.

V. NUMERICAL APPROXIMATIONS

In this section I will use an idealized sphere with similar dimensions as Earth so we can use it as reference for calculations, this sphere is a homogeneous, constant density, non rotating sphere with following characteristics:

k=6.9×10^{-10} m2/N [8]
ρ = 5510 Kg/m3
Radius=6370 Km

And we will use the following reference numbers:

G=$6.674\times10^{-11}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$
cylinder radius R=1mm
ε=10^{-10} meters (as a reference of inter-atomic distance in solids)
m (hypothetical) = 1Kg

Such sphere is under hydro-static equilibrium before the DM particle intersects the NM object, for which we can use the previous differential equation taking into account only the underlying sphere at r (see [10] for sphere is hydro-static equilibrium)

$$P_i(r)=-\frac{3GM^2}{8\pi R^6}(R^2-r^2)=-\frac{2G\rho^2\pi}{3}(R^2-r^2)$$

Can DM particle be trapped by Earth?

To simplify we will assume the DM particle crosses the sphere passing through its center.

Since we are setting the hypothetical mass of the DM particle to 1Kg the equation of the dynamical friction can be obtained from kinetic and potential energy variations:

$$vdv+\frac{4}{3}\pi\rho G r\,dr\approx-\pi k\rho G P_i(r)R_{cyl}\,dr=.$$
$$.=\pi k\rho G\frac{2G\rho^2\pi}{3}(R^2-r^2)R_{cyl}dr;$$
$$vdv\approx\pi\rho G\left(k\frac{2G\rho^2\pi}{3}(R^2-r^2)R_{cyl}-\frac{4}{3}r\right)dr$$

We can check what would be the incoming velocity so once the DM particle crosses the sphere its velocity is below the escape velocity (11 Km/s).

So integrating between vo and vf, and between r=-R and r=+R we get:

$$\frac{v_f^2-v_o^2}{2}\approx\pi\rho G k\frac{2G\rho^2\pi}{3}\left(2R^3-2\frac{R^3}{3}\right)R_{cyl}=.$$
$$\frac{8}{9}\pi^2\rho^3G^2kR^3R_{cyl}$$

which for a cylinder affected of 1mm radius we get a value of 7.66 $\times10^{-11}$ which is quite small to trap any DM particle that is not already gravitationally bounded to the planet.

Internal circular DM trajectory

If the DM particle is already gravitationally bounded inside a solid sphere of constant density and for simplicity it is following a circular orbit we can calculate how much the orbit will be shrinking due to dynamical friction.

Following a circular orbit of a radius "r" we know:

$$P_i(r)=-\frac{2G\rho^2\pi}{3}(R^2-r^2)$$

And the circular motion equation inside the planet:

$$\frac{4}{3}\pi\rho G r=w^2r;$$
$$w=\sqrt{\frac{4}{3}\pi\rho G}$$

so there is a fixed angular speed independent of the radius which for our NM sphere is

w=1,24x10^{-3} rad/seg or a period of 1,4 hours

We can then apply the differential formula for a small cylinder (torus in this case) around the DM particle trajectory

$$v\,dv+\frac{4}{3}\pi\rho G r\,dr\approx-\pi k\rho G P_i(r)R_{cyl}\,dl;$$

$$w\,r\,dr+\frac{4}{3}\pi\rho G r\,dr\approx-\pi k\rho G P_i(r)R_{cyl}\,r\,d\theta;$$

$$\frac{dr}{d\theta}\approx\frac{-\pi k\rho G P_i(r)R_{cyl}}{w+\frac{4}{3}\pi\rho G};$$

$$\frac{dr}{d\theta}\approx\frac{-\pi k\rho G R_{cyl}}{w+\frac{4}{3}\pi\rho G}\frac{2G\rho^2\pi}{3}(R^2-r^2)$$

in which with the given values we get:

$$\frac{dr}{d\theta}\approx 1.79\,10^{-28}(R^2-r^2);$$

$$\frac{dr}{1.79\,10^{-28}(R^2-r^2)}=d\theta;$$

$$\frac{1}{1.79\,10^{-28}}\left(-\frac{1}{2r}(\ln(\frac{R}{r}+1))-\ln(\frac{R}{r}-1)\right)\Bigg|_{10^6}^{10^6-1}=\theta\Big|_0^{\theta_{end}}$$

solving we get $\theta_{end} = 7.31\cdot10^{18}$ radians or what is equivalent $1.26\cdot10^{18}$ full orbits, each of a period of 1.4 hours it will require $1{,}86\cdot10^{14}$ years or more than 13 thousand times the age of the universe just for the orbit to drop 1m.

VI. CONCLUSION:

I have shown that DM particles should follow non-keplerian orbits when crossing NM objects, also given that those objects are not uniform the orbits will not be closed, although limited between a maximum and minimum radius. Articles that triggered this paper [2] and [3] should be revisited and corrected.

I have also shown that DM should loose energy when crossing NM objects and that an initial estimate of that shows a very small number when crossing earth, making it impossible to capture DM particles or even to reduce the size of the orbit once they have been trapped.

Similar estimates could be done with stars in which higher density, escape velocity and radius could allow for certain DM particles to be captured inside.

VII. ACKNOWLEDGMENTS AND DISCLAIMER

This paper was initially triggered by an article in “starts with a bang” in Forbes [2] which gave me initial discomfort in some of the sentences, nevertheless I have learned a lot from the articles by Ethan Siegel so I would like to thank him for his work in making hard science reachable to general public.

I have done some search in Arxiv (https://arxiv.org/search/advanced?advanced=&terms-0-operator=AND&terms-0-term=Dark+Matter&terms-0-field=all&terms-1-operator=AND&terms-1-term=Orbit&terms-1-field=title&classification-physics=y&classification-physics_archives=astro-ph&classification-include_cross_list=include&date-filter_by=all_dates&date-year=&date-from_date=&date-to_date=&date-date_type=submitted_date&abstracts=show&size=50&order=-announced_date_first) or Google about this topic and haven’t found any article about these two particular topics.

As of version 3 of this paper I acknowledge the Doctoral Thesis by Marina Cermeño Gavilán titled “Dark matter in dense astrophysical objects”, although the topic of the Thesis covers other scenarios, that is the first academia paper found by the author on the topic.

Nevertheless, it is very likely that this topic has already be presented before this paper, should that be the case, I recognize credit of it to whoever did it before me and I present apologies to not referencing it in this version.

Additionally, I have based the model on a limited set of information (mostly Wikipedia), Nicholas Wheeler Central Force Probleams, Ethan Siegel, and Wolfgang Cristian for the central force simulator [9] which I used and modified to show 3rd diagram, I would like to thank all the authors for the time taken to provide such useful information.

I would also like to thank my wife for her patience when I dedicate time for these ideas and my daughters for the notebook where I started sketching this idea.

I would also like to thank my brother Enrique Alonso and my cousin Guillermo Padilla as unaware reviewers of this first version paper once I publish it (note: this version is not reviewed).

I would also like to thank my sisters Rosella Alonso and Claudia Alonso, and my cousin Alba Gonzalez as they have also become non-volunteered reviewers of this paper second version. The excuse of having to print it or having to finish other things first is already been taken by the first two reviewers so the author expects innovative and imaginative excuses from them.

VIII. - REFERENCES

[1]https://www.forbes.com/sites/startswithabang/2019/02/15/cold-dark-matter-is-heated-up-by-stars-even-though-it-cannot-feel-them/?sh=8783883f1149

[2]https://www.forbes.com/sites/startswithabang/2017/11/02/what-would-happen-if-you-became-dark-matter/?sh=14679ab3fd29

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[4]https://en.m.wikipedia.org/wiki/Shell_theorem

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